Questions (or as we say, roadmap):

1. what does it mean for a system to be deterministic?
2. Does determinism imply Predictability?
3. Can't Predict? → is it indetermined?
4. what's chaos?
5. chaos = randomness?
6. can a deterministic system be chaotic?
7. unpredictability & Nature? → QUANTUM
8. unpredictability vs. determinism...

9. OK good! what about free will?

1. Determinism: $\longrightarrow$ Predictability?

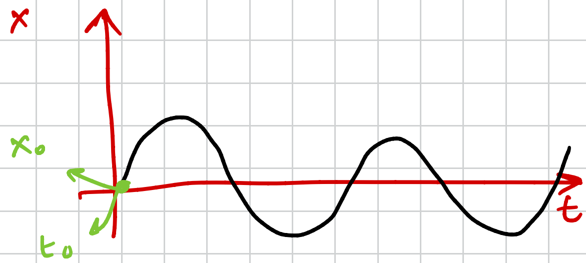
State at t_1 uniquely determines its state at $t_2 > t_1$, given the laws governing the system

$$|\psi(t_0)\rangle \ + \ \text{laws of evolution } \hat{U}(t) \ = \ |\psi(t)\rangle$$

$\Rightarrow$ only one possible trajectory from initial state..

ex1 Harmonic oscillator:

governing rule: $F = ma \xrightarrow[\text{for } k]{\text{for } k} \frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \xrightarrow[x_0 \checkmark]{t_0 \checkmark} x(t) = x_0 \cos(\omega t - \omega t_0) + \frac{V_0}{\omega}\sin(\omega t)$



predictability?

Laplace's demon: if we know
- all the forces acting on nature (laws)
- the x, p of every objects at t_1

$\Rightarrow$ we can calculate all past & futures (just like Planets.)

Deterministic law + complete knowledge of $x(0)$, $P(0)$ $\Rightarrow$ Predictable future

Q: Does Determinism imply predictability?

- Determinism: the future is uniquely fixed (determined) by the present states & the Deterministic laws!
- Predictability: An observer $\hat{A}$ can determine what that future state will be.

(Not logically equivalent)

Here comes Poincaré!

3-body gravitational Problem:

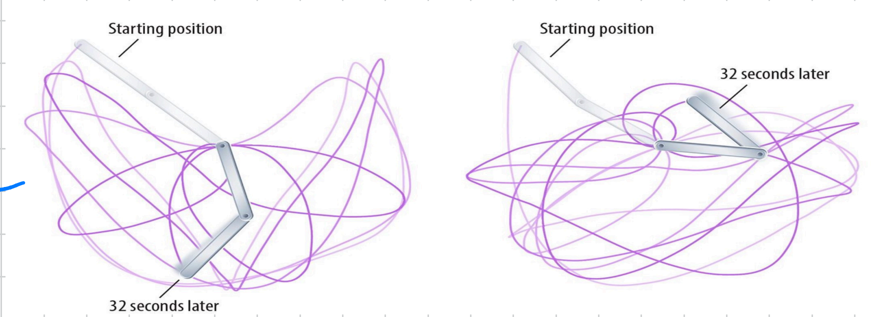
* the laws are still deterministic and no random parameter...

but the dynamic is super-sensitive on initial conditions

$$\begin{cases} A : x_0 \\ B : x_0 + \epsilon \end{cases} \Rightarrow |x_A^0 - x_B^0| \approx 0 \xrightarrow{\hat{U}(t)} |x_A(t) - x_B(t)| = \text{large}!$$

two completely different trajectories!

both trajectories are individually deterministic...



$\Rightarrow$ tiny unc in IC $\longrightarrow$ large unc in future!

Seed of chaos Theory

Mitchell: classical assumption that deterministic laws automatically guarantee long-term predictability begins to break down by Poincaré's results!

Poincaré did not say:
- The system is random (what's random?)
- The system is indeterministic -X-

$$\boxed{\text{Deterministic} \overset{?}{\not\Rightarrow} \text{Predictable}}$$

Some extra quotes (maybe good for questions:)

- Determinism is a statement about how the system evolves!
- Predictability is a statement about our ability to determine the future!

I guess the problem that distinguish these two, is that physical measurements always have finite precision!

$\boxed{x_0 = 0.20000000}$ $\xrightarrow{\text{measurement}}$ $\boxed{0.1999999 < x_0 < 0.2000001}$

$\longrightarrow$ chaos : $\delta(t) \approx \delta_0 e^{\lambda x}$ .... $\Longrightarrow$ macroscopic unc.

I guess the unpredictability is not intrinsic, it's due to small unc that gets amplified in certain dynamical systems.

Mitchell : A complete det equation can generate behavior that appears random

~~Randomness ≠ unpredictability~~ , ~~chaos ≠ randomness~~

Q: what's unpredictable? The system or our knowledge??

Q: weather example? → Lorentz → Butterfly effect?

Superposition:

$$f(ax_1 + bx_2) = af(\cancel{a}x_1) + f(bx_2)$$

$$= af(x_1) + bf(x_2)$$

$$f(x) = 2x$$

Superposition consists Homogeneity + additivity

x^2, x^3

$$f(x) = x^2$$

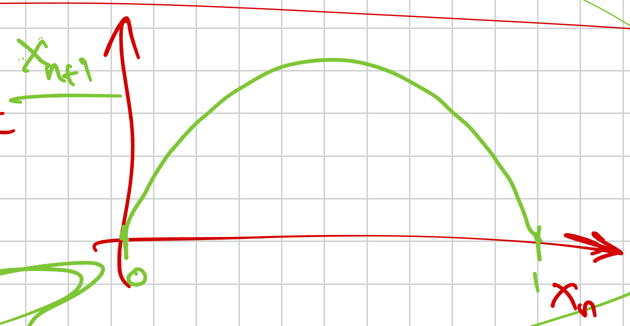
interaction

$$f(x_1+x_2) = (x_1+x_2)^2 = x_1^2 + 2x_1 \cdot x_2 + x_2^2$$

Cross. term

$$f(x_1+x_2) = x_1^2 + x_2^2$$

$$x_{n+1} = R\,x_n(1-x_n) =$$



2. chaos: why it behaves like that?

Nonlinearity:

→ linearity: $X_{t+1} = a X_t$

initial: $x_0' = x_0$, $x_0'' = x_0 + \delta$

1 ⇒ $|x_1'' - x_1'| = a\delta_0$

⇒ $\delta^n = |x_n'' - x_n'| = a^n \delta_0$

$\delta_0 \to 0 \Rightarrow \delta^n \to 0$

→ nonlinear: $\frac{dx}{dt} = ax(1-x) = ax - ax^2$

$\Rightarrow \boxed{X_{t+1} = a X_t (1 - X_t)}$ → logistic map!

see the video ...

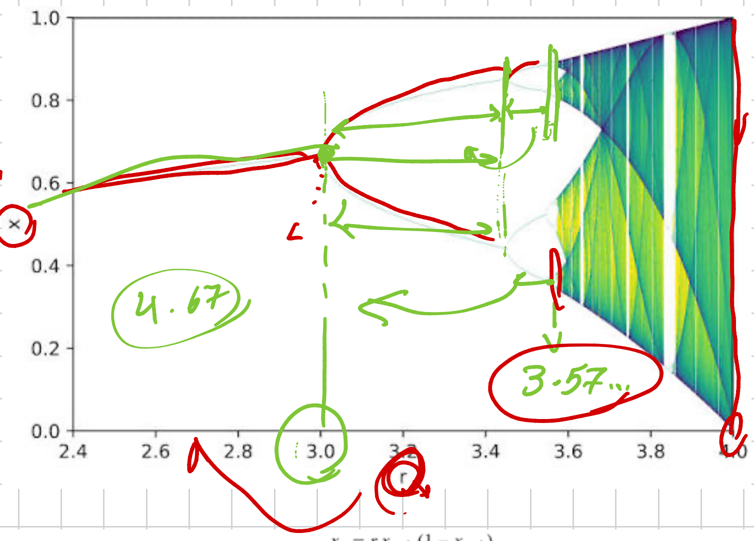
$ax_t - ax_t^2$

Not all the nonlinear systems are chaotic! → in logistic map, depending on

a → stable, periodic, or chaotic behavior!

↳ controlling the dynamic...

equilibrium position



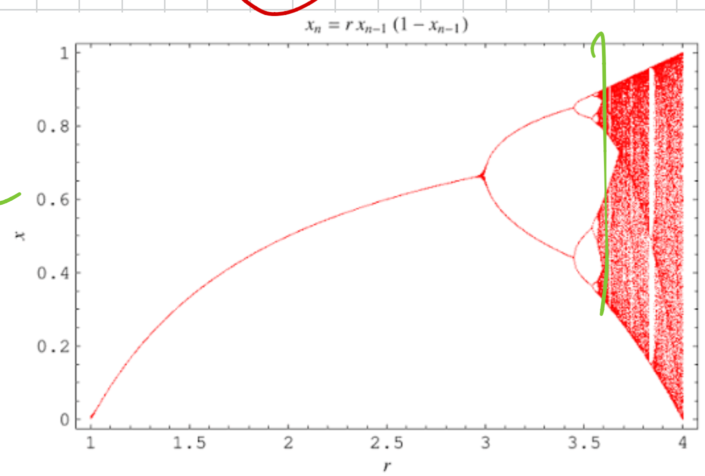
Same equation, Same det rule,

→ Increasingly complex dynamics

↓

* go to the back to show the diagrams & chaotic behavior

bifurcation



Page 30 - 32 complexity... →

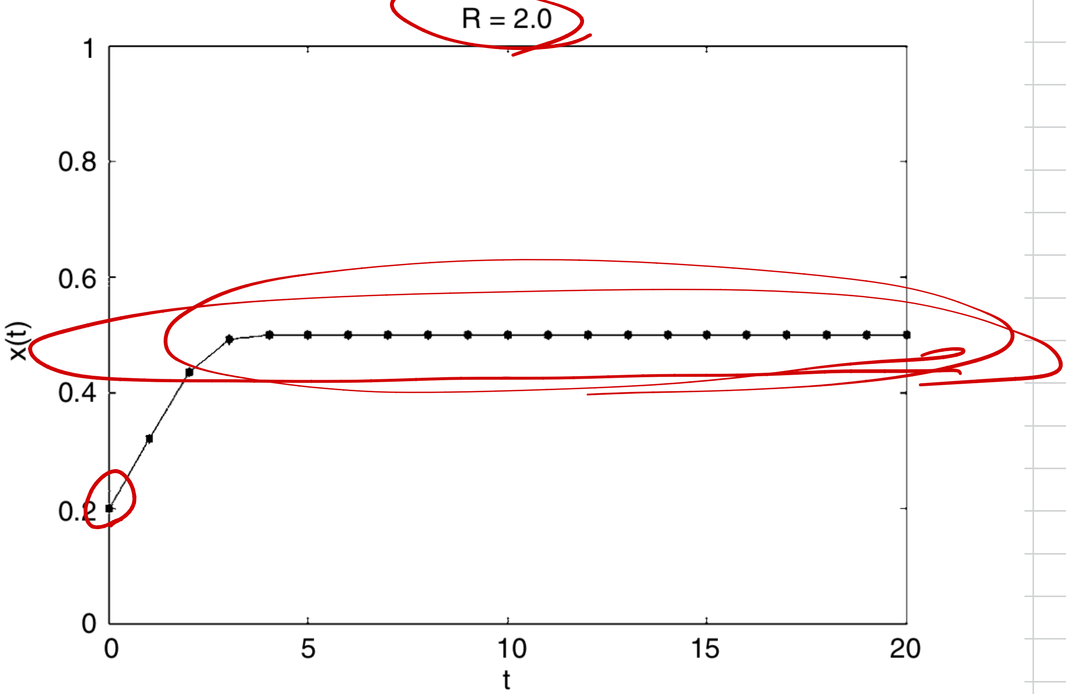


FIGURE 2.6. Behavior of the logistic map for $R = 2$ and $x_0 = 0.2$.

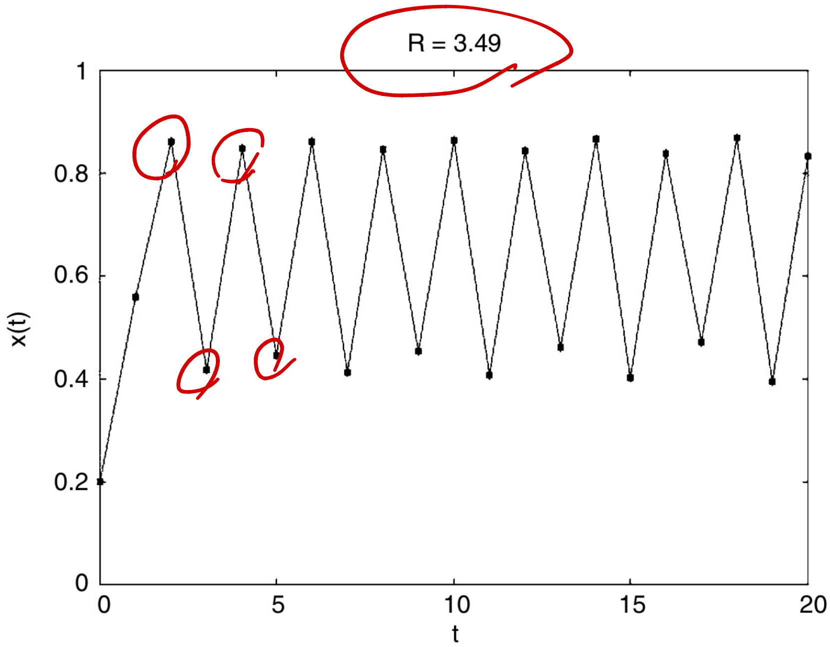


FIGURE 2.9. Behavior of the logistic map for $R = 3.49$ and $x_0 = 0.2$.

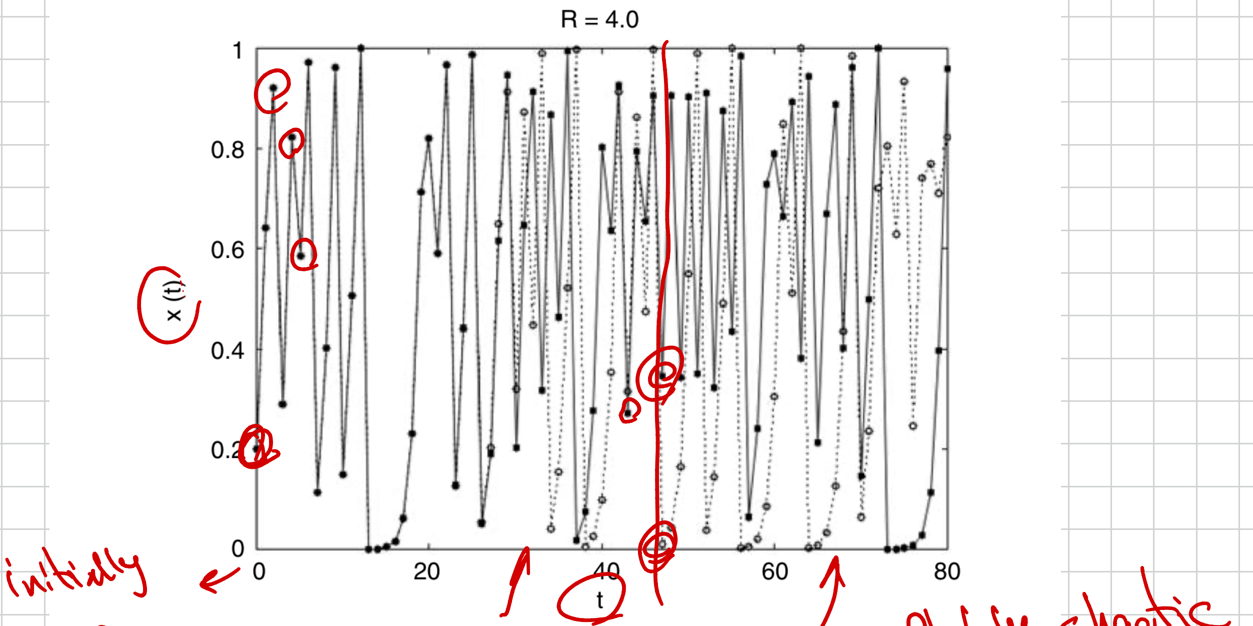


FIGURE 2.10. Two trajectories of the logistic map for $\boxed{R = 4.0}$, $x_0 = 0.2$ and $x_0 = 0.2000000001$.

BTW, Randomness (Pseudo) can arise from det systems!

Robert May, the mathematical biologist, summed up these rather surprising properties, echoing Poincaré:

> The fact that the simple and deterministic equation (1) [i.e., the logistic map] can possess dynamical trajectories which look like some sort of random noise has disturbing practical implications. It means, for example, that apparently erratic fluctuations in the census data for an animal population need not necessarily betoken either the vagaries of an unpredictable environment or sampling errors: they may simply derive from a rigidly deterministic population growth relationship such as equation (1). . . . Alternatively, it may be observed that in the chaotic regime arbitrarily close initial conditions can lead to trajectories which, after a sufficiently long time, diverge widely. This means that, even if we have a simple model in which all the parameters are determined exactly, long-term prediction is nevertheless impossible.

In short, the presence of chaos in a system implies that perfect prediction *à la* Laplace is impossible not only in practice but also *in principle*, since we can never know x_0 to infinitely many decimal places. This is a profound negative result that, along with quantum mechanics, helped wipe out the optimistic nineteenth-century view of a clockwork Newtonian universe that ticked along its predictable path.

Chaos: to describe dynamical systems with super-sensitivity dependence on initial condition!

Q: if infinite precision, would chaos appear?

The issue is that the actual physical observer cannot possess infinite precision!

Sapolsky: Chaos can undermine our ability to reconstruct or predict a system without undermining determinism itself.

* there is some order in chaos!

Revolutionary ideas from chaos:

- Seemingly ~~random behavior can emerge from deterministic systems, with no external source of randomness.~~
- The behavior of some simple, deterministic systems can be impossible, *even in principle*, to predict in the long term, due to sensitive dependence on initial conditions.
- Although the detailed behavior of a chaotic system cannot be predicted, there is some "order in chaos" seen in universal properties common to large sets of chaotic systems, such as the period-doubling route to chaos and Feigenbaum's constant. Thus even though "prediction becomes impossible" at the detailed level, there are some higher-level aspects of chaotic systems that are indeed predictable.

4.669

Q: what Poincaré couldn't solve?

Poincaré could not find a general analytical solution to the three-body Problem.

for two body gravitational Problem: $m_1 \ddot{r}_1 = G m_1 m_2 \frac{r_2 - r_1}{|r_2 - r_1|^3}$

$\Rightarrow r(\theta) = \frac{P}{1 + e\cos\theta}$ → general analytic solution

but for 3-body ⇒ No general closed form → only numerical –

No $r(\theta)$ simple formula for each object...

4. Reductionism

Q: if we cannot predict the behavior of a system from its microscopic state, can we still say that its behavior is determined by its microscopic components?

Red vs. Cons:

Red: Can we understand the whole sys by reducing it to its components & their interactions?

→ explained

whole → Part

why does a gas have pressure?

Don't treat pressure as a fundamental mysterious property, understand the molecules, their $\vec{v}$, collisions & interactions

why does that person decide to do X?

Decision → neural circuit → neurons → synaps → moleculs → etc...

Con: Can we construct complex behavior from simple components and simple rules?

→ generate whole by parts...

Parts + rules → whole

To be continued by Sapolsky's book & lecture

Determined   stanford